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The Quincunx as an Educational Tool

By: Steve Moore

Sir Francis Galton's invention can do more than just demonstrate normal distribution.

The purpose of this article is to give you an appreciation of the Quincunx as an educational tool for teaching some of the theory behind the tools and concepts of so-called modern quality management. The Quincunx is often seen in the possession of organizations practicing in-house education of statistical process control (SPC); however, it is seldom utilized for anything beyond a demonstration of the normal distribution. Indeed, the literature itself is virtually devoid of references to the Quincunx beyond this use.

Sir Francis Galton (1822–1911) invented the Quincunx in the 1870s to demonstrate the law of error and the normal distribution. He was a prolific inventor, scientist, and mathematician and was knighted in 1909. Beyond the Quincunx, Galton conceived the standard deviation, invented the use of the regression line, and was the first to describe the phenomenon of the regression toward the mean.

Description of the Quincunx

The Quincunx Model WD-7 (see figure 1) is an example of a an off-the-shelf Quincunx board and is comprised of a vertical board with 10 rows of pins. Beads are dropped via a funnel into the top of the board. As they descend through the board the beads will bounce either to the left or right as they encounter each row of pins. The pins may be arranged such that a bead will come in contact with 4, 6, 8, or 10 rows. As each bead leaves the final row of pins, it is captured in one of several bead-wide bins which may be numbered for reference by the user. After a sufficient number of beads have been dropped, the height of the beads in the bins begins to resemble the classic bell-shaped curve. In reality, the distribution of beads is binomial; however, the normal distribution is approximated when n, the number of rows of pins, is large (n = 4, 6, 8, or 10 for the Quincunx referred to in this article).

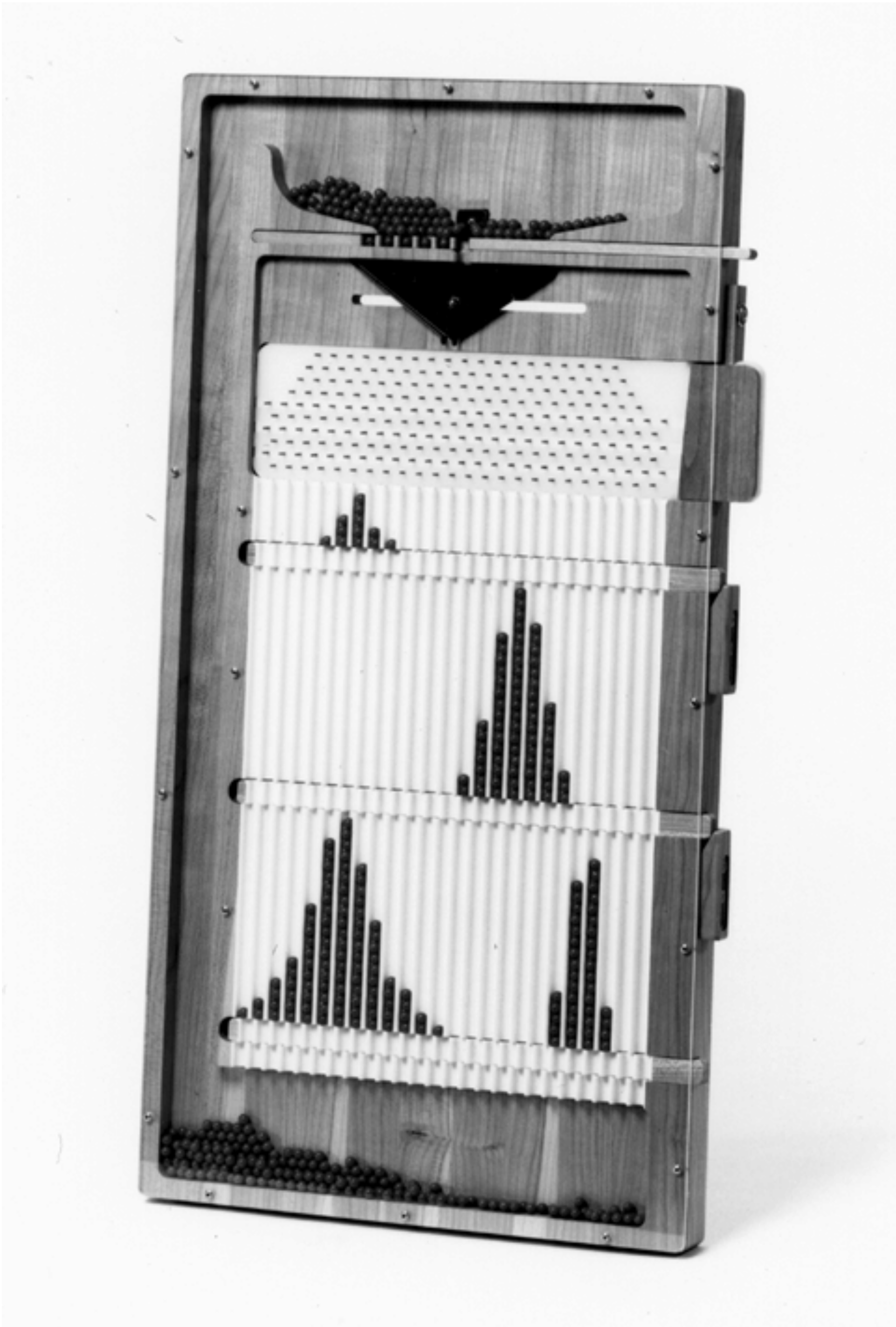

Figure 1: Quincunx Model WD-7 (Courtesy of Lightening Calculator, www.qualitytng.com [1])

When a bead is dropped, it will bounce to the right from zero to k times (and to the left for the remaining pins). It then lands in the kth bin. The bins may be numbered zero to k either from left to right or right to left. The number of paths a bead can take to land in the kth bin is given by the standard binomial coefficient for n choices taken k at a time.

Quincunx and process behavior charts

There is much literature that contains the misleading notion that data must be normally distributed for a process behavior chart (control chart) to work. Quincunx data is actually binomially distributed, especially as n decreases toward four (the minimum for the Quincunx model used here). As seen in figures 2 and 3, XmR control charts for 100 bead drops when n=4 and n=10 are both quite successful. Note that, as expected, the control limits are farther apart as n increases because the distribution will be wider as the beads contact more rows of pins.

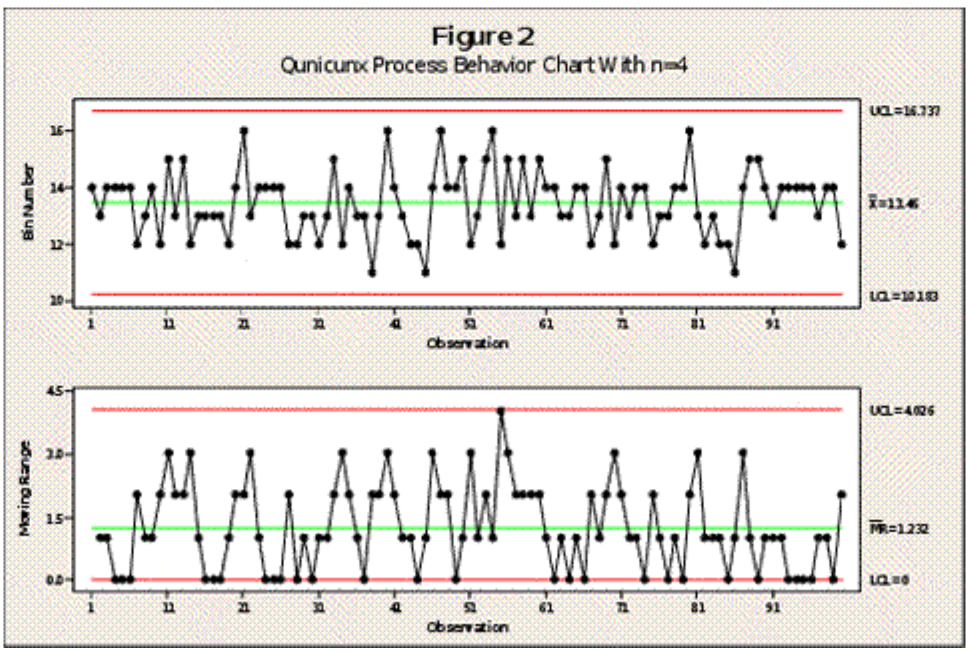

Figure 2: Quincunx process behavior chart with n=4

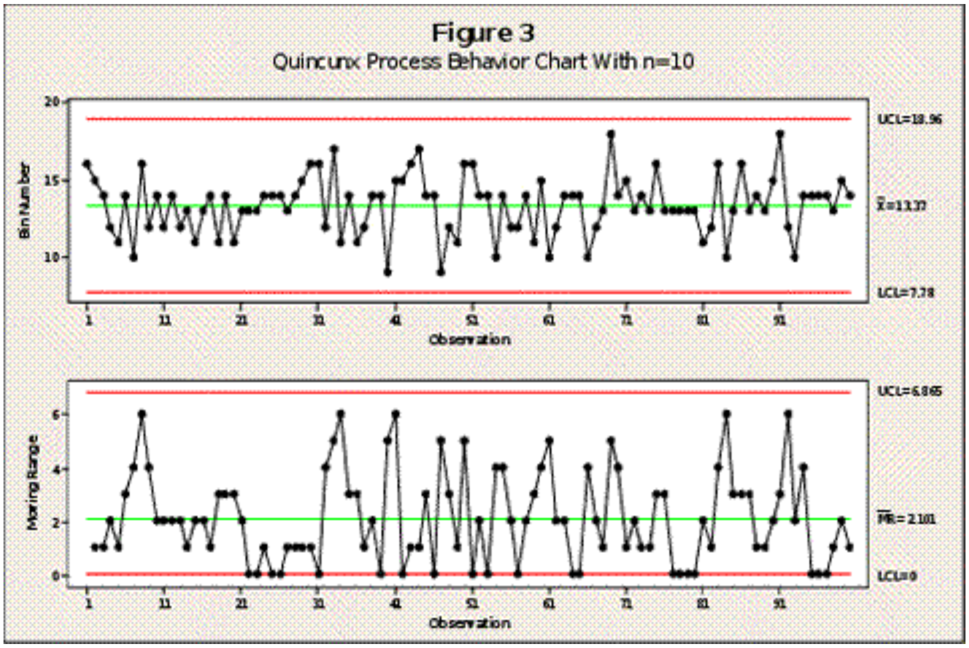

Figure 3: Quincunx process behavior chart with n=10

As a further demonstration of the fallacy of the normal distribution requirement, an experiment was performed with the Quincunx to obtain a significantly skewed distribution. This was achieved by placing a vertical barrier of heavy card stock strips in the pins so that no bead could travel to the left more than twice. Then 100 beads were dropped and the results plotted on an XmR control chart. The histogram and resulting control charts are seen in figures 4 and 5. They clearly demonstrate that the chart is fully functional in spite of being far from normally distributed.

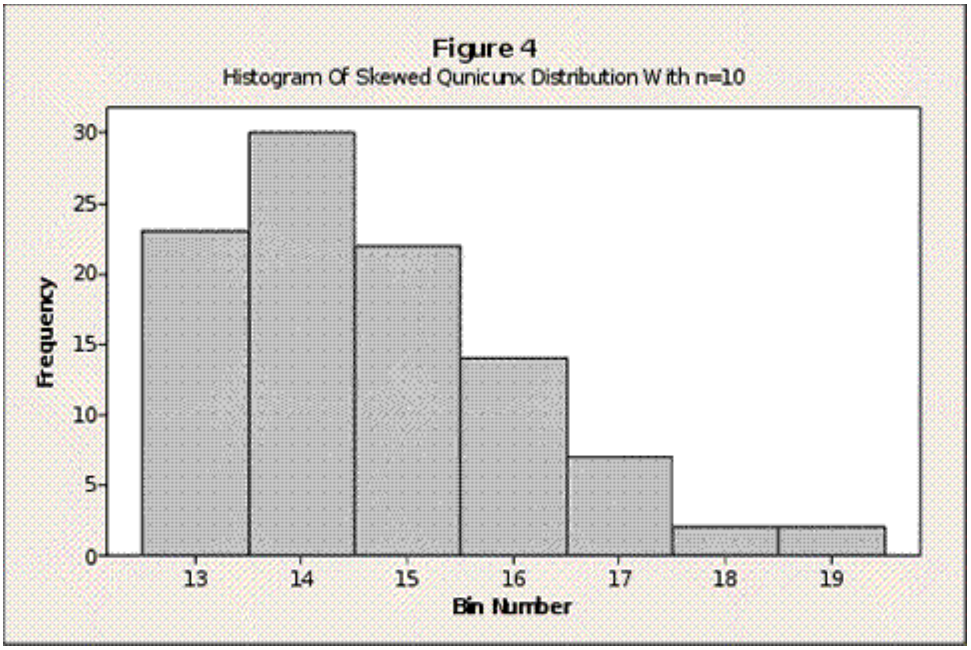

Figure 4: Histogram of skewed Quincunx distribution with n=10

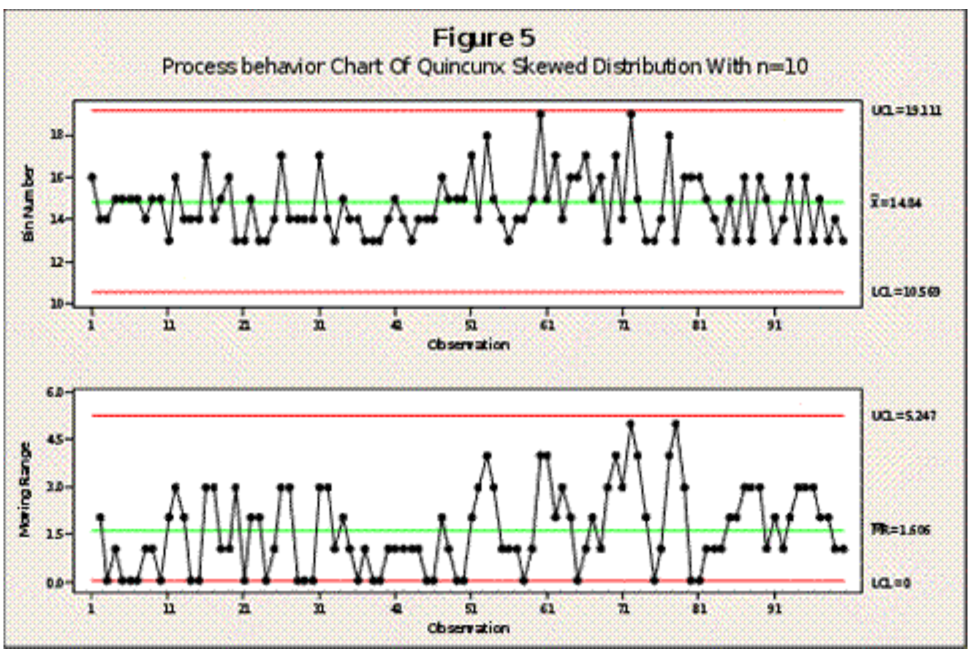

Figure 5: Process behavior chart of Quincunx skewed distribution with n=10

Another area of dispute regarding control charts is the number of data required for control limits to be useful. Figure 6 shows the results of an experiment in which control limits of an XmR chart were calculated after every bead was dropped, beginning with the fourth bead and continuing until the fortieth. The upper control limit quickly stabilizes around a median of 17.52 after the ninth bead is dropped. This demonstration dispels the rumor that at least 20 to 30 data points are needed to calculate useful control limits and confirms the Donald Wheeler arguments.

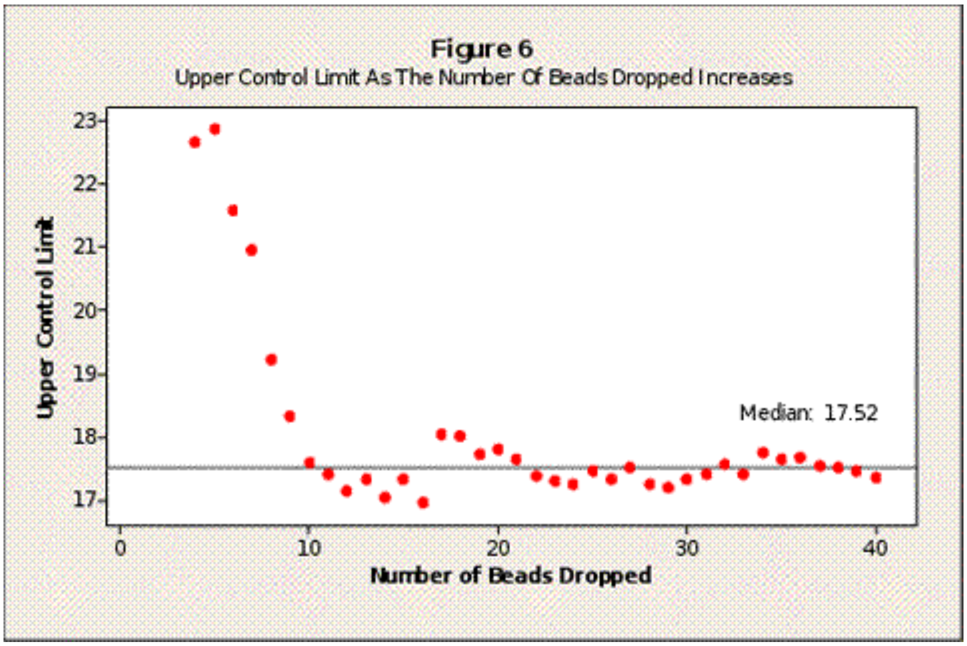

Figure 6: Upper control limit as the number of beads dropped increases.

I have successfully utilized the Quincunx to teach the principles of SPC to “students” from the shop floor to upper management for many years. I have used two major exercises: A demonstration of “tampering,” and a demonstration of a stable process when a change has been made to the system.

Tampering

The lesson of tampering is a good first introduction to the Quincunx. When introducing it to students, have the front panel over the funnel and pins covered with a piece of paper or cardboard so that they cannot be seen. Allow the students to gain tribal knowledge as they experiment and speculate as to the functions of the funnel and pin settings. The students are then instructed to “aim” at a particular bin and are allowed to move the funnel and pin settings even though they are not quite sure of the structures under the covered panel. After 50 trials, note the shape of the distribution of the beads about the target bin. This distribution will tend to be a uniform distribution as the number of trials becomes large.

Next, place the funnel directly above the targeted bin and set the pins so that n=4. After 50 beads have been dropped, compare the new distribution of beads with the previous one. The lesson will be clear: Leaving the process alone will be significantly more accurate and precise than tampering with the aim.

The lesson on tampering can be extended to demonstrate the four rules of W. Edwards Deming’s famous Funnel Experiment. Figures 7, 8, and 9 are process behavior charts demonstrating Rules 2, 3, and 4, respectively with n=10. Figure 3 demonstrates Rule 1. It is instructive to note the change in the control limits from those generated by Rule 1 as Rules 2 and 3 are invoked. The process behavior chart for Rule 4 shows a complete lack of statistical control.

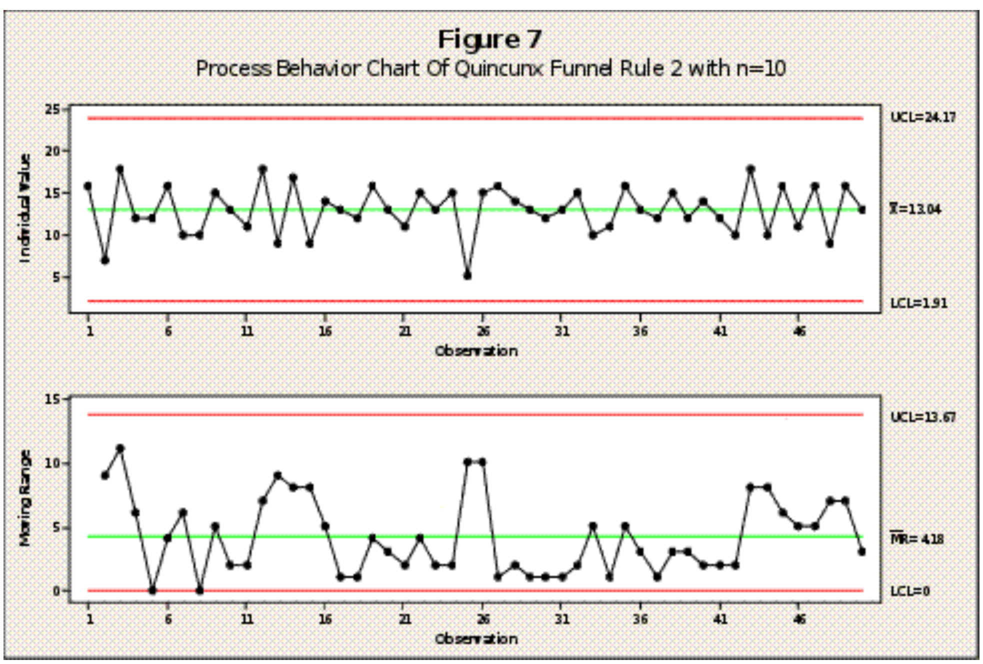

Figure 7: Process behavior chart of Quincunx funnel rule 2 with n=10

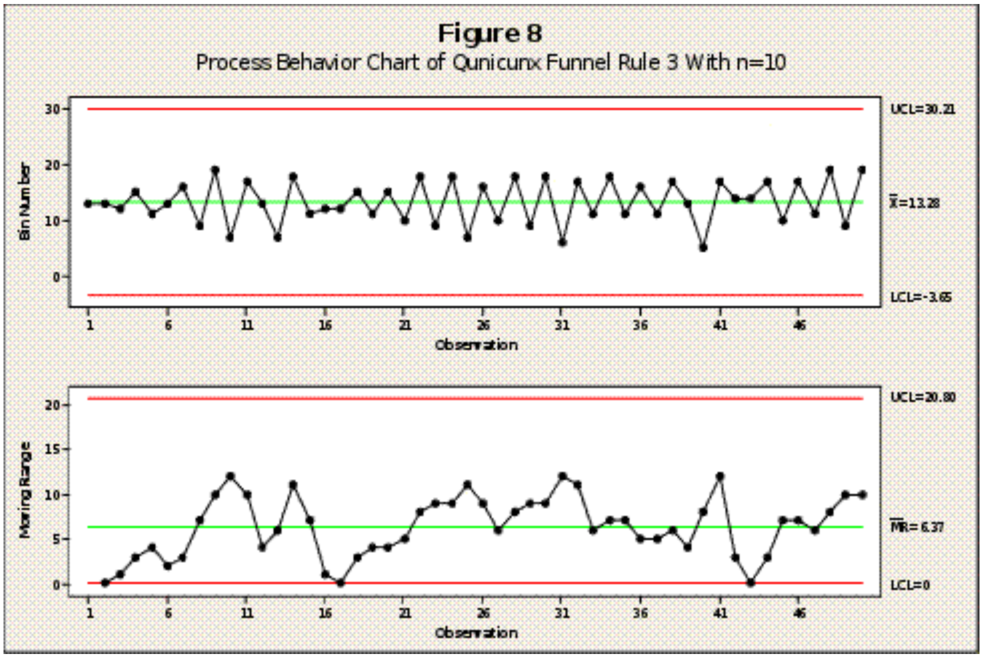

Figure 8: Process behavior chart of Quincunx funnel rule 3 with n=10

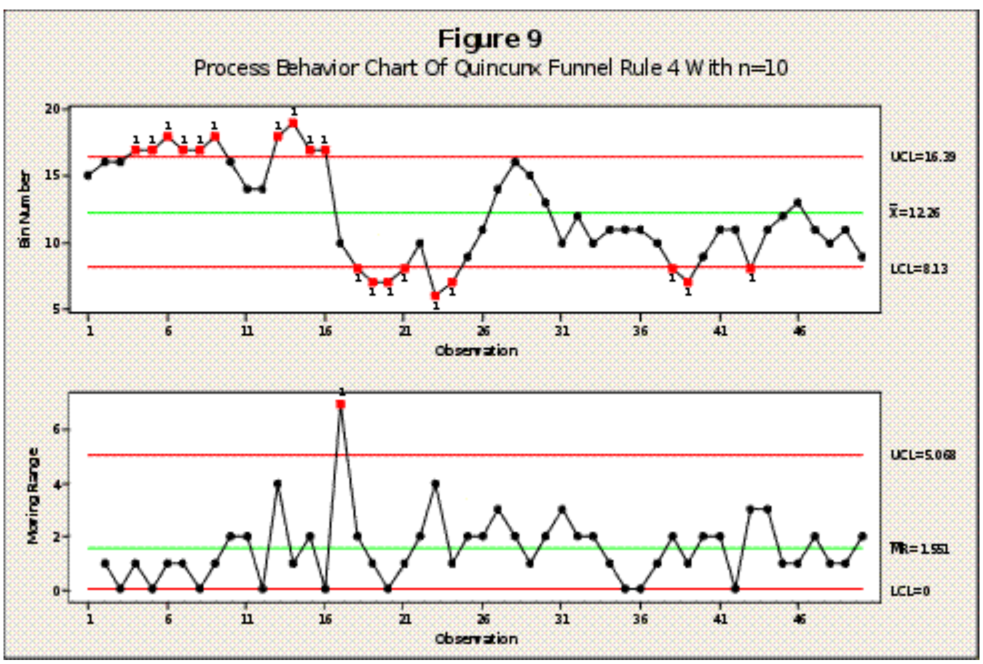

Figure 9: Process behavior chart of Quincunx funnel rule 4 with n=10

Learning about process behavior charts with a Quincunx

With the funnel held stationary and n=10, have students drop 50 beads and construct an XmR chart of the numbered bins that the beads fall into. Note the upper and lower control limits and discuss the sources of variation. Next, move the funnel 1.5 pins to the left or right and take 10 more data points. The control chart will quickly show one or more signals of lack of control against the original control limits as seen in Figure 10. This demonstration may also be performed with an X-R bar chart, but this, of course, requires more class time.

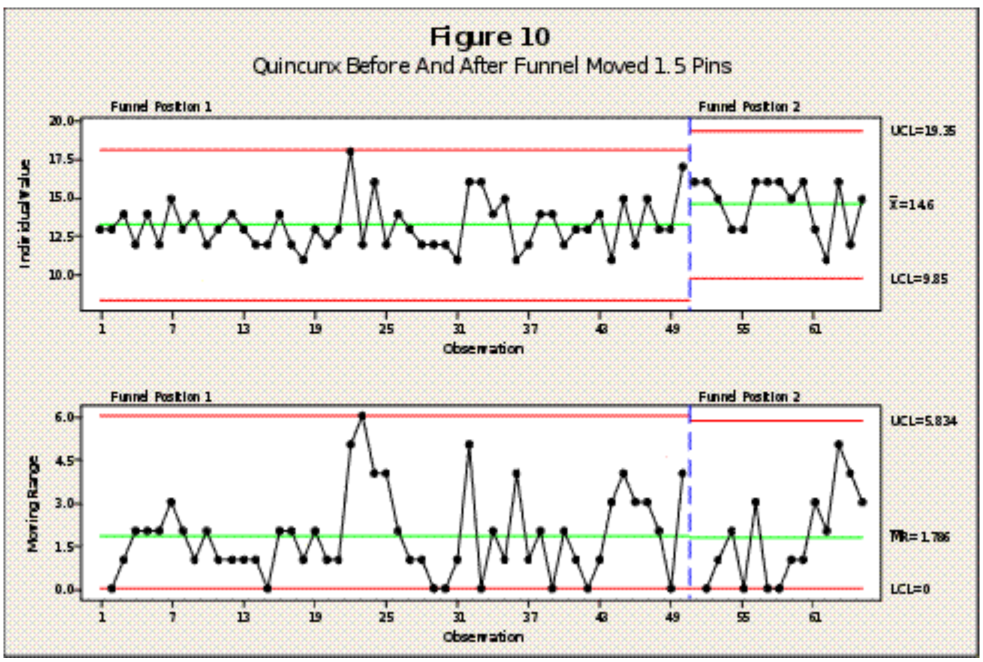

Figure 10: Quincunx before and after funnel moved 1.5 pins.

Summary

The Quincunx is a valuable learning tool that has been mostly overlooked in classroom settings. With a little imagination, this tool can be effectively utilized to teach many lessons in the understanding of variability and process behavior charts.

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